Alternate Current Dynamo-Electric Machines

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VI. *Alternate Current Dynamo-Electric Machines.*

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The paper deals experimentally with the current induced in the coils and in the cores of the magnets of alternate current machines by the varying currents and by the varying positions of the armature. It is shown that such currents exist and that they have the effect of diminishing to a certain extent the electromotive force of the machine when it is working on resistances as a generator without having a corresponding effect upon the phase of the armature current. It is also shown that preventing variations in the coils of the electromagnet does not, in the machine experimented upon, greatly affect the result, and that the effect of introducing copper plates between the magnets and the armature has not a very great effect upon the electromotive force of the armature, the conclusion being that the conductivity of the iron cores is sufficient to produce the main part of the effect. A method of determining the efficiency of alternate current machines is illustrated and the results of the experiments for this determination are utilised to show that in certain cases of relation of phase of current to phase of electromotive force the effect of the local currents in the iron cores is to increase instead of to diminish the electromotive force of the machine.

I.

In algebraic discussions of the theory of alternate current machines, it has usually been assumed that the electromotive force due to the magnets is a periodic function, the same whether there is a current in the armature or not, and that the effect of the current in the armature can be represented by regarding the armature as having self-induction. It has been pointed out, too, that the coefficient of self-induction will generally vary with the position of the armature in the field. To state exactly the same thing in another way it has been assumed that the electromotive force of the magnets is a periodic function independent of the current in the armature, and that the effect of the armature current on the induction through the armature can be

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* The large majority of the experiments herein described were made in the summer of 1893 and a considerable part of the paper was then written. We have to thank Mr. F. Lydall, one of the student demonstrators at King's College at that time, for much assistance.

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represented as an armature reaction which vanishes at the moment when the current in the armature vanishes.

To state the matter in the form of an equation, let E be the electromotive force of the machine on open circuit, R the resistance of the armature circuit, x the current in the armature, T the periodic time, then it is assumed that

$$\mathrm{R}x = \mathrm{E} - (\mathrm{L}x)^{\cdot}\,;$$

E being independent of x, L being, if you please, a coefficient of self-induction constant or variable, or, if you prefer it, Lx representing the change in the induction through the armature due to the current in the armature, vanishing with x.

It is easy to see that this statement is true in some cases. For example, it is very nearly true in the older machines with permanent magnets. Or imagine a machine without iron in the magnets or armature, consisting merely of two circuits—one the magnet circuit, the other the armature circuit—movable in relation to each other. If the current in the magnet circuit is kept precisely constant, either by inserting a great self-induction in its circuit external to the machine, or by inserting such a resistance and using so high an electromotive force that any disturbing electromotive forces are inappreciable compared with it, the preceding statement is strictly accurate. But if the magnet current is not forced to be constant the problem is more complicated.

Stating the matter in the language of self- and mutual-induction, let x and y be the currents in armature and magnet circuits, R and r their resistances, L and N their self-induction, M sin $2\pi t/\mathrm{T}$ their mutual-induction, F the constant electromotive force applied to the magnet, the equations for the system are :—

$$\left.\begin{aligned} \mathrm{R}x &= -\,\mathrm{M}\,(y \sin 2\pi t/\mathrm{T})^{\cdot} - \mathrm{L}x^{\cdot} \\ ry &= \mathrm{F} - \mathrm{M}\,(x \sin 2\pi t/\mathrm{T})^{\cdot} - \mathrm{N}y^{\cdot} \end{aligned}\right\}$$

These equations can be solved by approximations if the variations in the value of y are small.

First,

$$y = \frac{\mathrm{F}}{r}\,; \qquad x = -\,\frac{\mathrm{MF}}{r}\,\frac{2\pi}{\mathrm{T}}\,\frac{\mathrm{R}\cos 2\pi t/\mathrm{T} + 2\pi\mathrm{L}/\mathrm{T}\sin 2\pi t/\mathrm{T}}{\mathrm{R}^2 + (2\pi\mathrm{L}/\mathrm{T})^2}\,.$$

Second, substituting this value of x, we obtain

$$ry = \mathrm{F} + \frac{\mathrm{M}^2\mathrm{F}}{r}\cdot\frac{2\pi/\mathrm{T}}{\mathrm{R}^2 + (2\pi\mathrm{L}/\mathrm{T})^2}\left\{\frac{\mathrm{R}}{2}\sin\frac{4\pi t}{\mathrm{T}} + \frac{\pi\mathrm{L}}{\mathrm{T}}\left(1 - \cos\frac{4\pi t}{\mathrm{T}}\right)\right\}^{\cdot} - \mathrm{N}y^{\cdot}.$$

This gives periodic terms in y, the period being one-half the period of the mutual induction.

Introducing these terms into the first equation, we see that the term in $2\pi t/\mathrm{T}$ in the electromotive force of the machine is modified, and that terms in $6\pi t/\text{T}$ are introduced. The former may have real practical importance, and it is one of the objects of the present paper to ascertain how far it exists and is of importance in actual machines.

Returning to machines as ordinarily constructed, in these the current in the magnet coils is not compelled to be constant, and any rapid variation of the induction in the magnet cores will induce currents in those cores. The variations in the current in the magnet coils and the currents in the cores both tend to annul the variations in the induction in the core arising from the current in the armature, but they will not tend to alter the *average* effect of the currents in the armature on the induction through the magnets. That there will be such an average effect is not difficult to see. Suppose the armature coils to be fixed in line with the magnets of the machine, any current in the armature will have its full effect in increasing or diminishing the field through the magnets. Suppose the armature coils to be fixed midway between the magnets, any current in the armature will then have practically little or no effect in increasing or diminishing the field through the magnets. If the armature be connected through resistance, inductive or non-inductive, and the machine run in the ordinary way, the current in the armature will lag in phase behind the electromotive force E. The result is that when the armature is opposite to the magnets there is a current in the armature tending to demagnetize the magnets, and adding together the effects of the armature in all positions, there is an average effect tending to demagnetize the magnets. If the machine had a constant current round the magnets and divided iron in the magnets, this average effect, as well as its variations, would be fully accounted for by the term $(\text{L}x)^{\cdot}$, call it self-induction or call it armature reaction, as you please. But inasmuch as the variations are in part annulled by the variations of current in the magnet-winding and the local currents in the magnet cores, we have a part of the diminution of the electromotive force E of the machine unaccompanied by retardation of phase of the current in the armature.

Before giving any experimental results, it will be well to describe the machines used and the general method of experiment adopted.

The two dynamos experimented upon were constructed by Messrs. SIEMENS BROS., and are of the same pattern and size but are of an old type. They are mounted upon a common base-plate, their pulleys being provided with flanges and bolts, so that any desired phase difference can be given to the armatures, for the accurate setting of which a graduated circle is provided, or so that the armatures can be run independently of each other. The pulleys have each a diameter of 12 inches, and are suited for a 6-inch belt—the shaft is prolonged for the purpose of carrying a revolving contact-maker and a small pulley for driving a Buss tachometer.

Each dynamo has a series of 24 electromagnets (see fig. 1), there being 12 on either side of the armature. The core of each magnet (A) is of wrought-iron $2\frac{1}{16}$ inches diameter, and $6\frac{1}{4}$ inches long: and is wound with 5 layers, 35 convolutions per layer, of copper wire 3·5 millims. diameter. The electromagnets are bolted to circular cast-iron frames (B), which serve also for supporting the bearings of the armature shaft. The centres of the 12 electromagnets on each frame are equally spaced out on a circle $8\frac{7}{8}$ inches radius concentric with the axle of the machine. Each cast-iron frame has a cross-sectional area of 4·8 sq. inches. The opposing pole pieces of the electromagnets have an air space of $1\frac{1}{8}$ inch between them, through which the armature coils rotate. The 24 electromagnet windings are coupled in series, and have a total resistance of 1·8 ohms, the normal exciting current being about 22 amperes.

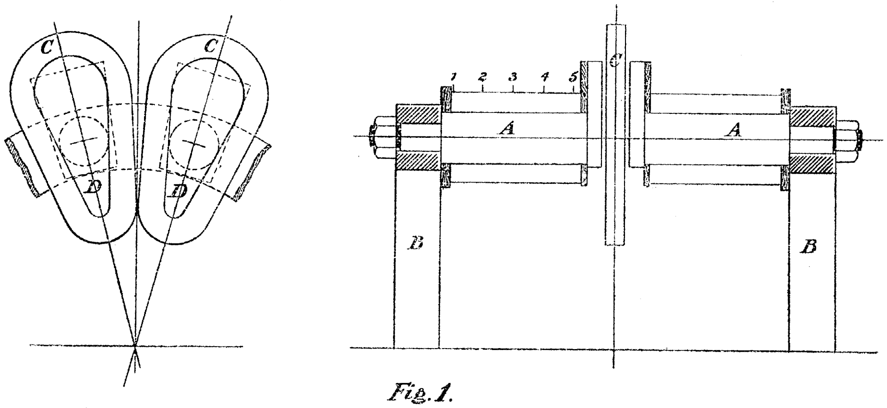


Fig. 1.

The armature of each dynamo consists of 12 coils or bobbins (C) with wooden cores (D) $\frac{7}{8}$ inch thick. Each core is 7 inches long (radially), with rounded ends—the outer being struck to a circle $3\frac{1}{8}$ inches and the inner $1\frac{1}{8}$ inches diameter. The ends of the respective coils are brought to a screw-plug commutator board fixed to the shaft, by means of which a series of combinations can be made. Each coil consists of 10 layers, 8 convolutions per layer, 2·2 millims. copper wire, having a resistance (cold) of ·187 ohm, and at a speed of 1000 revolutions per minute, with normal excitation and fully loaded, is intended to give 50 volts at its extremities. The full load current for each coil is 16·6 amperes, so that at 1000 revolutions per minute, or a frequency of 100 complete periods per second, the machine should give, with its 12 armature coils in parallel, an output of 200 amperes 50 volts. The two terminal rings of the screw-plug commutator are connected by conductors to two gun-metal collector rings, insulated from one another and from the shaft by means of ebonite. Each collector ring is provided with two 1-inch copper wire brushes, carried by adjustable bar-holders fixed to the terminal blocks of the dynamo.

The potential difference between any two points at any epoch is determined by means of a Kelvin quadrant electrometer and a revolving contact-maker fixed to the shaft of the dynamo. The contact-maker consists of a disc of gun-metal which carries two rings, one of gun-metal insulated from the disc, the other of ebonite. Into the latter is inserted a strip of metal $\frac{5}{64}$ inch wide, which is in permanent contact with the gun-metal ring. Two insulated brushes are attached to a movable brush-holder, so that one presses on each ring. The circuit between the two brushes is completed once in each revolution, and the position of the contact can be read off by a pointer attached to the holder on a circle $13\frac{1}{2}$ inches diameter, divided into 360 degrees. The two points between which it is desired to measure the potential difference are connected through the contact-maker to a condenser and the quadrant electrometer, as shown in fig. 2, in which A and B are the points, C the revolving contact-maker, D the reversing switch of the electrometer, E the condenser, and F the quadrant electrometer. By plotting as ordinates the volts measured at any epoch, and as abscissæ the position of the contact-maker as representing time, the curve of potential is obtained. The electrometer is standardised by means of a Clark cell, so that the deflections on the scale can be reduced to volts: when the potential difference between A and B was too great for the electrometer, it was reduced in any desired ratio by two considerable non-inductive resistances introduced between A and B, as shown in fig. 3.

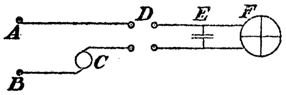


Fig. 2.

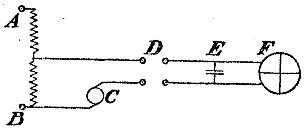
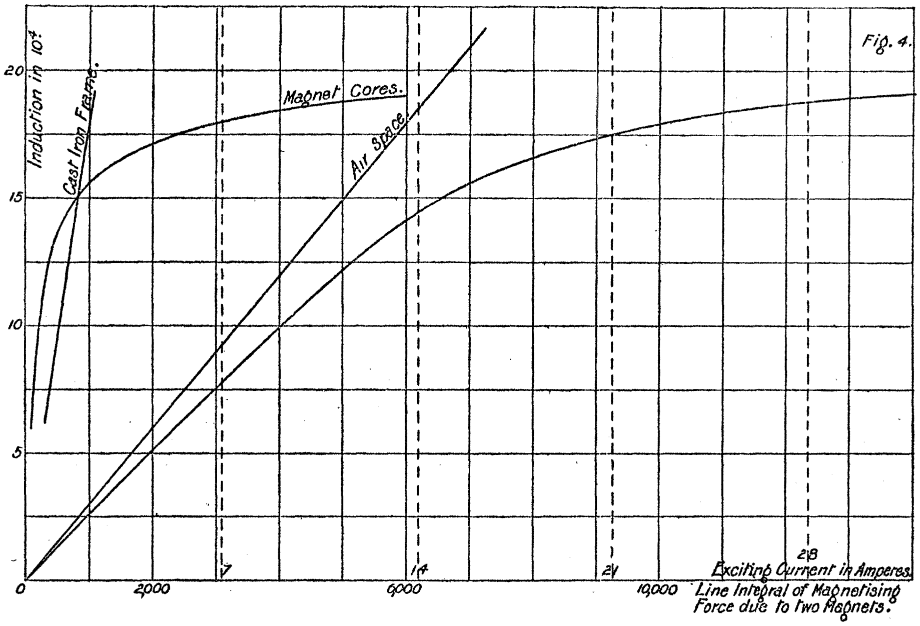


Fig. 3.

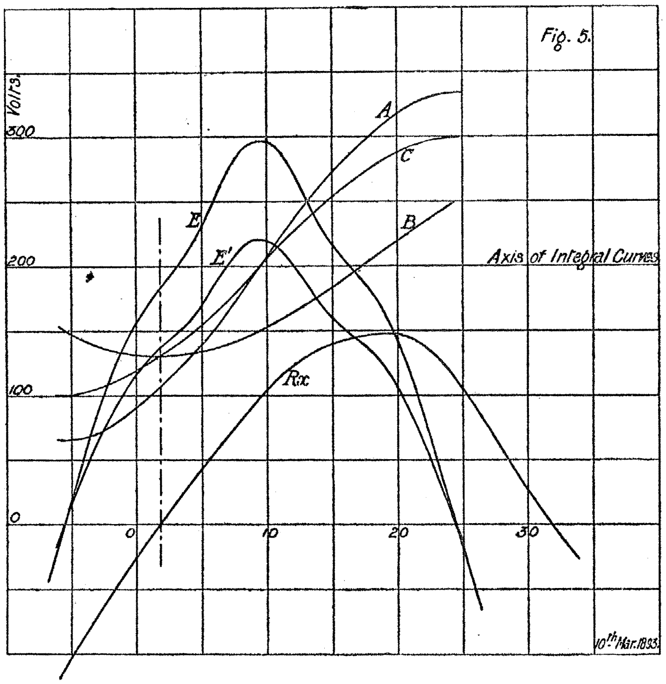
Fig. 4.



The characteristic curve of the alternator is given in fig. 4, and shows the relation between the total induction I, between one pole piece and the opposing one, in terms of the line integral of magnetising force due to the two windings in series on the two respective magnet cores; the scale of amperes in the magnet winding is also given horizontally.

In fig. 5 the speed of the alternator experimented upon was 923 revolutions per minute, or a frequency of 92·3 complete periods per second. For the purpose of obtaining a marked effect from the current in the armature a large current was taken out of the armature, and the current in the magnet winding was only 8 amperes. A Kelvin multicellular voltmeter placed across the terminals of the machine read 190 volts on open circuit, and 81 volts when loaded, and a Kelvin ampere balance in the external non-inductive circuit read 40 amperes. The armature bobbins were coupled 6 in series 2 parallel between the brushes, the total resistance of the circuit (R) was 2·52 ohms, the armature resistance alone being ·55 ohm.

Fig. 5.



Curve E is the electromotive force curve of the machine when there is no current in the armature. Rx is the curve of electromotive force deduced from the potential difference taken between the terminals of the alternator when supplying current through non-inductive resistances. The curves E and Rx have been integrated, the corresponding integral curves being A and B respectively.

That the ordinary theory does not fully account for the facts is easily shown. We have $\mathrm{R}x = \mathrm{E} - (\mathrm{L}x)^{\cdot}$. Integrate both sides from any fixed epoch 0 to any time t and we have

$$\int_0^t \mathrm{R}x\,dt = \int_0^t \mathrm{E}\,dt - \mathrm{L}\,(x_t - x_0).$$

Each term of this equation consists of a constant part and of a periodic part. The constant parts must be equal and also the periodic parts. We have to deal only with the periodic parts. The curves A and B, fig. 5, represent the periodic parts of each of the first two terms; the difference of ordinates of these curves should at all times be equal to Lx. In particular, and this is the only point of moment, as we do not know how L may vary, when Lx vanishes $\int \mathrm{R}x\,dt = \int \mathrm{E}\,dt$.

If the effect of the current in the armature on the induction through the armature could be represented by a term which vanishes with the current in the armature, it is obvious that the curves A and B would cross at the epoch when $x = 0$. They do not. The difference of inductions as given by these curves at this epoch is 25 per cent. of the induction which would *then* traverse the armature coil if the machine were running on open circuit, that is to say, 25 per cent. of the ordinate at this epoch given by the A curve. If it is assumed that the *average* effect of the armature current upon the induction in the magnets is such as to lower this induction by 25 per cent., the ordinates of the E curve would be decreased in like proportion, giving the curve E′ of which C is the integral. The difference at any epoch between the curve C and the Rx curve is to be fully accounted for by the term $(\mathrm{L}x)^{\cdot}$.

We may put it in this way. In this machine the armature current at the times when it has a value affects the field at the instant when the armature current is zero. The effect is produced by variations induced in the current in the field magnet-winding and in the solid iron of the magnets by the varying current in and the varying position of the armature. That these induced currents must exist is obvious, and it is easy enough to measure them in the copper coils. They have the effect of causing the armature reaction to produce an average effect upon the magnetism of the fields by partially annulling its periodic effect. If the current in the armature did not lag behind the electromotive force of the magnets E, there would be little or no diminution of the average magnetism of the magnets. We may correctly say that this diminution of the magnetism of the magnets is due to the self-induction of the armature causing a lag of current. The effect arises from the self-induction of the armature modified by currents induced in the magnets.

The effect on the magnets of any current in the armature is greatest when the armature bobbins are opposite the pole-faces, it is *nil* or small when half-way between the pole-faces. We may therefore represent its effect at any instant approximately as proportional to the expression

$$\frac{2\pi \mathrm{L}/\mathrm{T} \sin 2\pi t/\mathrm{T} + \mathrm{R} \cos 2\pi t/\mathrm{T}}{(2\pi \mathrm{L}/\mathrm{T})^2 + \mathrm{R}^2} \sin 2\pi t/\mathrm{T}$$

or

$$\frac{\pi \mathrm{L}/\mathrm{T}}{(2\pi \mathrm{L}/\mathrm{T})^2 + \mathrm{R}^2} - \frac{\pi \mathrm{L}/\mathrm{T} \cos 4\pi t/\mathrm{T} - \mathrm{R} \sin 4\pi t/\mathrm{T}}{(2\pi \mathrm{L}/\mathrm{T})^2 + \mathrm{R}^2}.$$

The constant term causes the fall in average magnetism, the periodic term causes currents in the magnets, and its effect on the magnetism is partly annulled thereby. The effect will vary as the square of the current if this is small. The effect of self-induction in diminishing the apparent electromotive force of the machine varies as

Fig. 6.

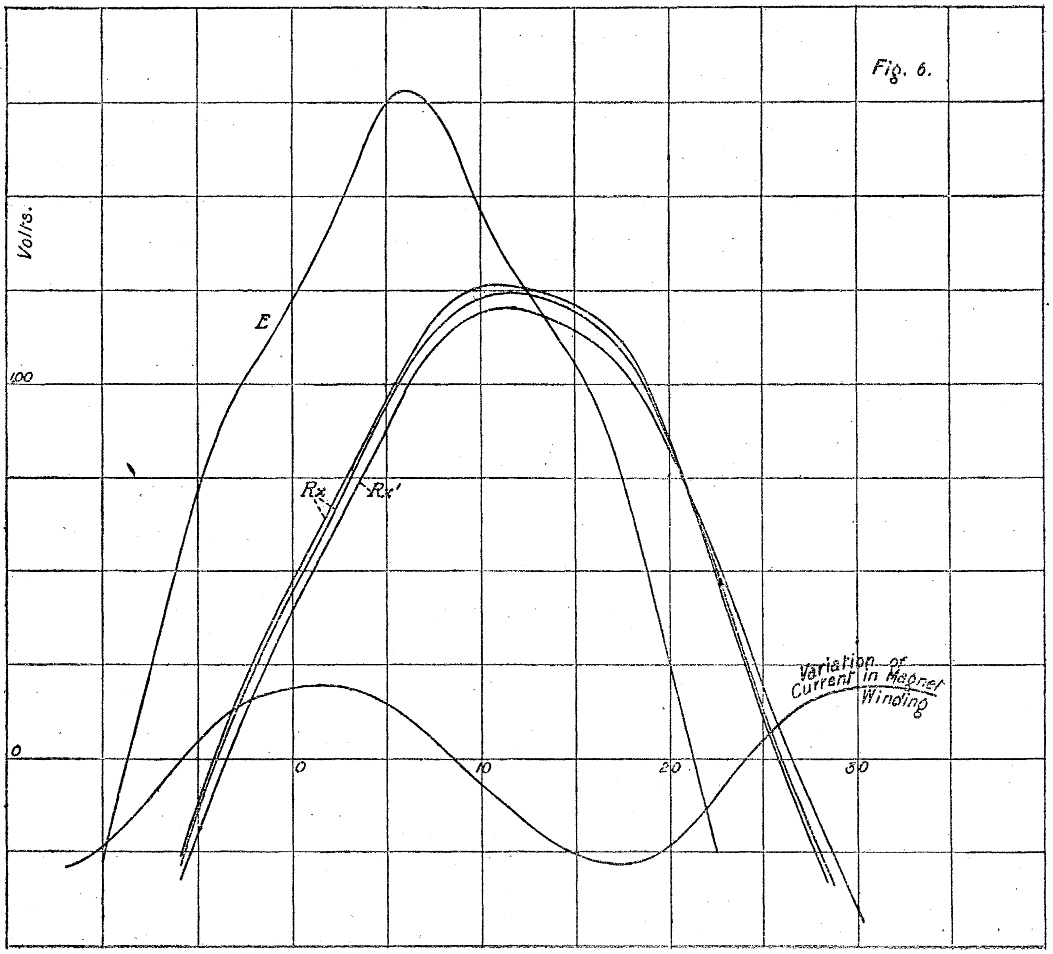
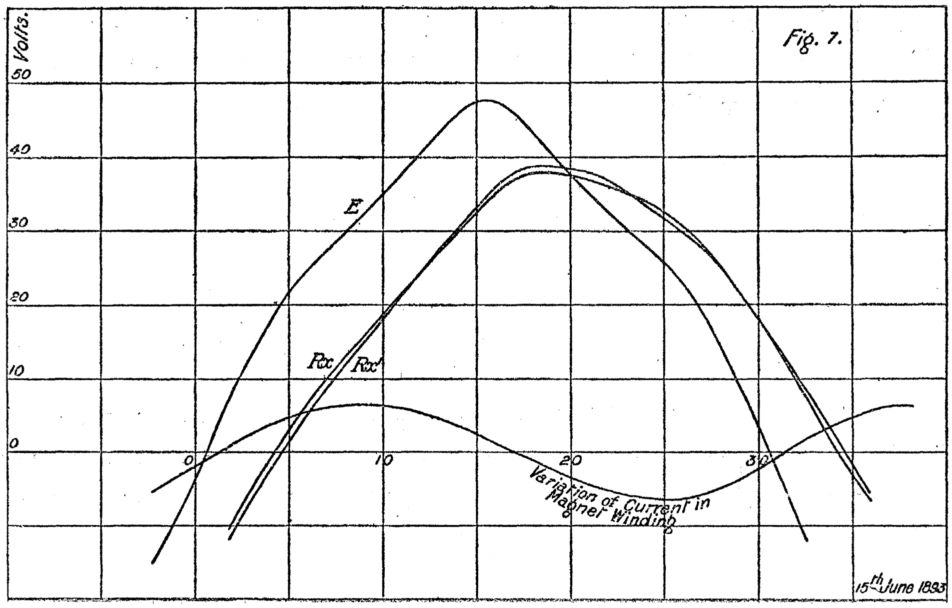


Fig. 6.
Volts.
100
0
E
Rx
Rx'
0
10
20
30
Variation of Current in Magnet Winding

Fig. 7.


Volts.
50
40
30
20
10
0
Fig. 7.
E
Rx
Rx'
0
10
20
30
Variation of Current in Magnet Winding
15th June 1893

the square of the current, so that we may expect the two effects, that due to the reaction of the armature on itself and that brought about by the armature inducing currents in the magnets, to vary together.

The lag of phase is less than we should expect from the diminution of electromotive force, or the electromotive force suffers greater diminution than we should expect from the angle of lag of phase.

We have tried a number of experiments for the purpose of tracing the variations of current in the magnets, and also with the intention of increasing or diminishing the effect we have observed. It is easy enough to trace the variations in the current round the magnets by measuring at points during the period the potential difference between the ends of a non-inductive resistance in series with the magnets. These variations are shown in figs. 6 and 7, in which the armature bobbins were coupled, 4 series, 3 parallel, and 12 series, respectively. We see, as we should expect, that the variations have a periodic time one-half the periodic time of the machine. But the current round the magnet could be made constant by exciting the two machines with the same current, loading each to the same degree, and placing their armatures one-fourth part of a period apart in phase.

The machines were run under conditions set forth in Table I., and the curves are given in figs. 5, 6, 7, 8. An electromotive force curve E, was observed with no current in the armature, and a curve of potential difference was taken between the brushes with the alternator loaded on a non-inductive resistance. Rx is this curve *with* variations in the exciting current, and Rx' *without* variations, in each case corrected for the resistance of the armature. Figs. 6 and 7 show the curves of actual electromotive force, Rx and Rx', when the current in the magnet winding is allowed to vary and when its variations are stopped. It will be seen that they do not differ a great deal. What currents are stopped in the magnet winding no doubt turn up in the substance of the cores themselves and have an effect not differing greatly. Curve E represents electromotive forces observed. *Rx* represents the potential difference taken between the brushes and corrected for the resistance of the armature, with the alternator working on a non-inductive resistance *with* variations in the exciting current. *Rx' without* such variations in each case. The induced currents are in either case adequate to nearly stop the variation of induction. Fig. 8 shows the same thing.

An exploring coil was wound and placed on one of the magnet limbs and the electromotive force in it was observed in terms of the time for the various positions of the exploring coil, marked 1, 2, 3, 4, 5, in fig. 1. Both the amplitude and the epoch varied with the position of the coil, but, in all cases, the periodic time was half the periodic time of the machine. It does not seem worth while to publish the curves connecting electromotive force and time.

Lastly, we tried to exaggerate the effects; for this purpose we introduced plates of copper, $\frac{1}{8}$ inch thick, in the form of two flat rings between the pole faces and the armature. Curves 9, 10, 11, 12, give the results for two different currents round

Fig. 8.

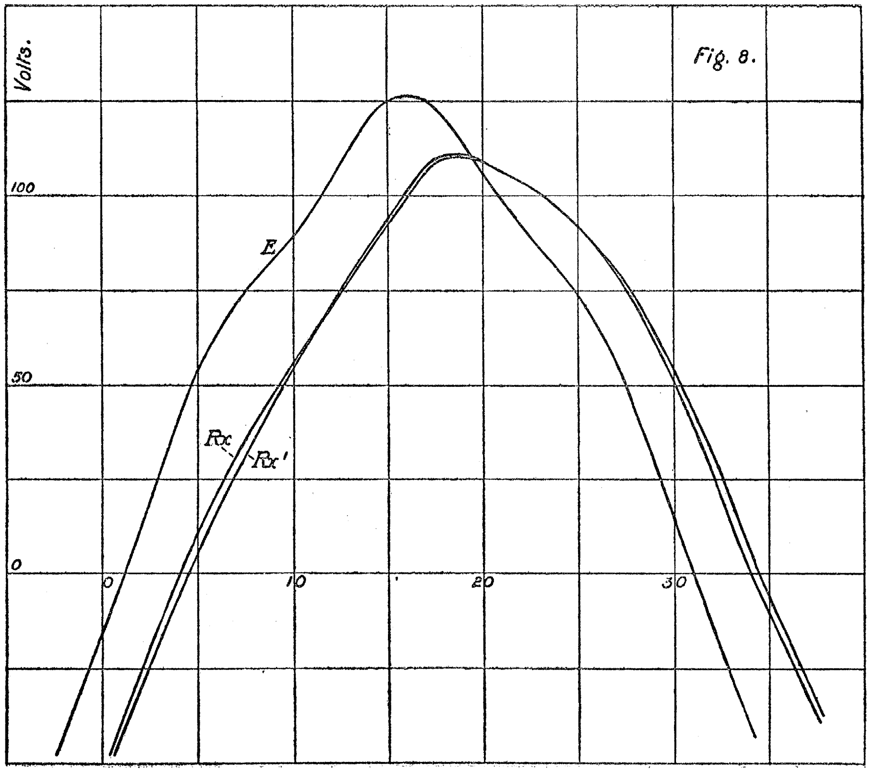
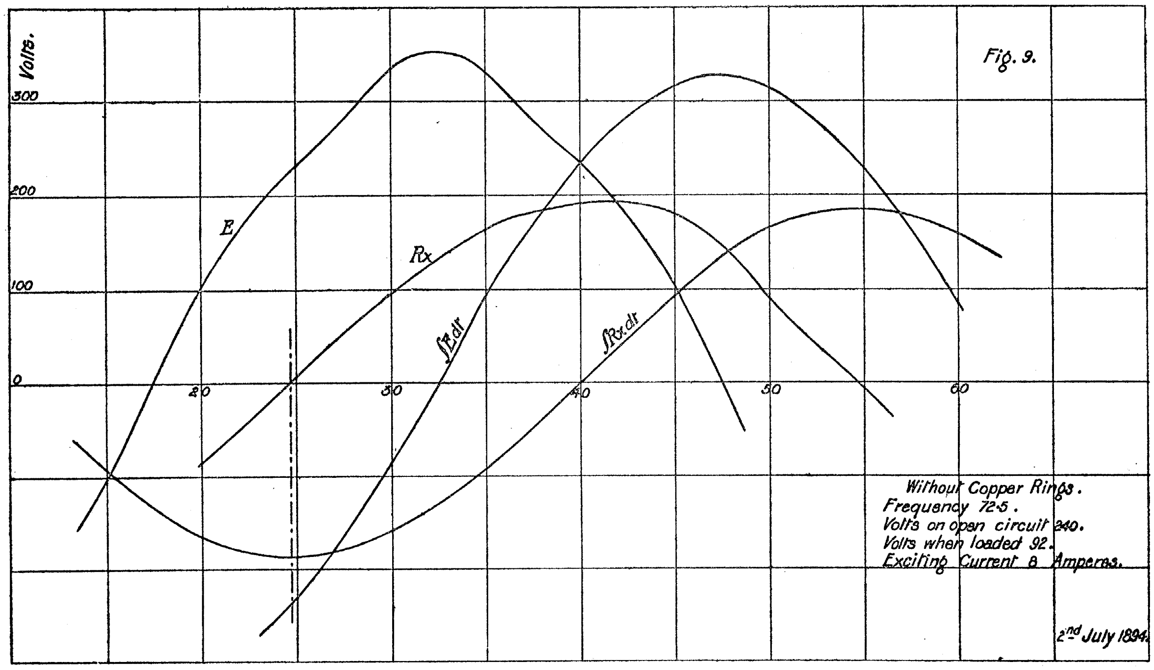


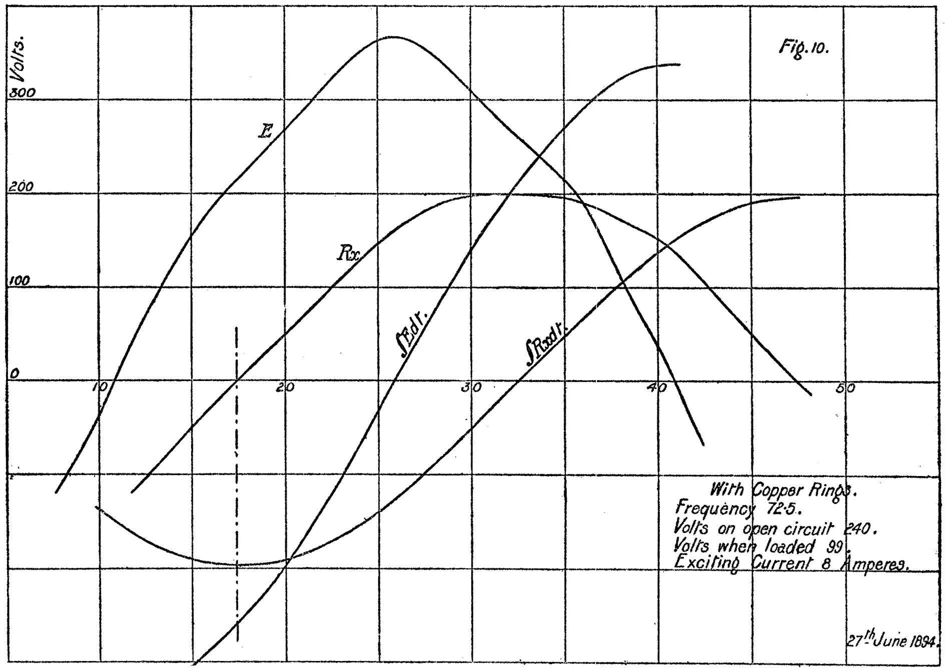
Fig. 9.



Without Copper Rings.
Frequency 72·5.
Volts on open circuit 240.
Volts when loaded 92.
Exciting Current 8 Amperes.

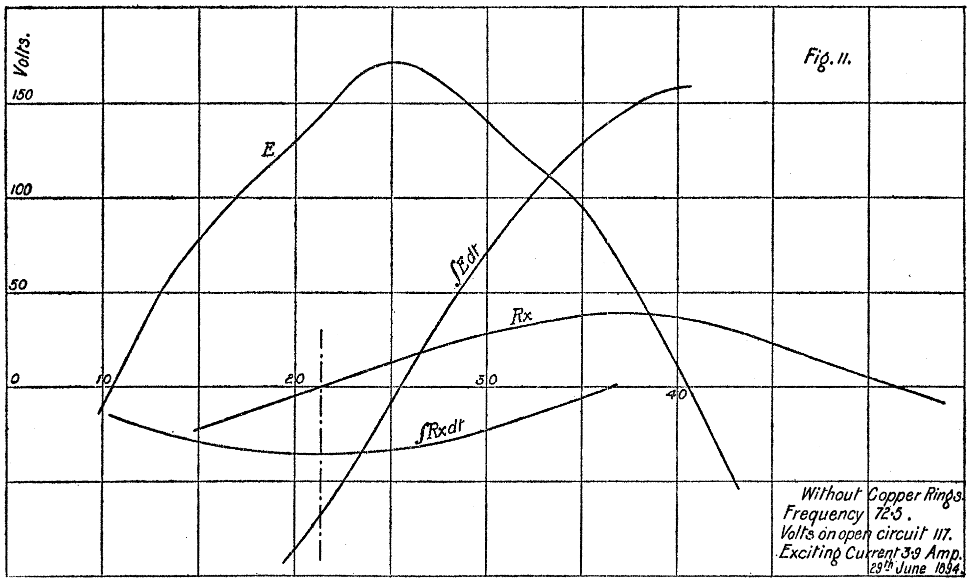
2nd July 1894.

Fig. 10.



With Copper Rings.
Frequency 72·5.
Volts on open circuit 240.
Volts when loaded 99.
Exciting Current 8 Amperes.
27th June 1894.

Fig. 11.



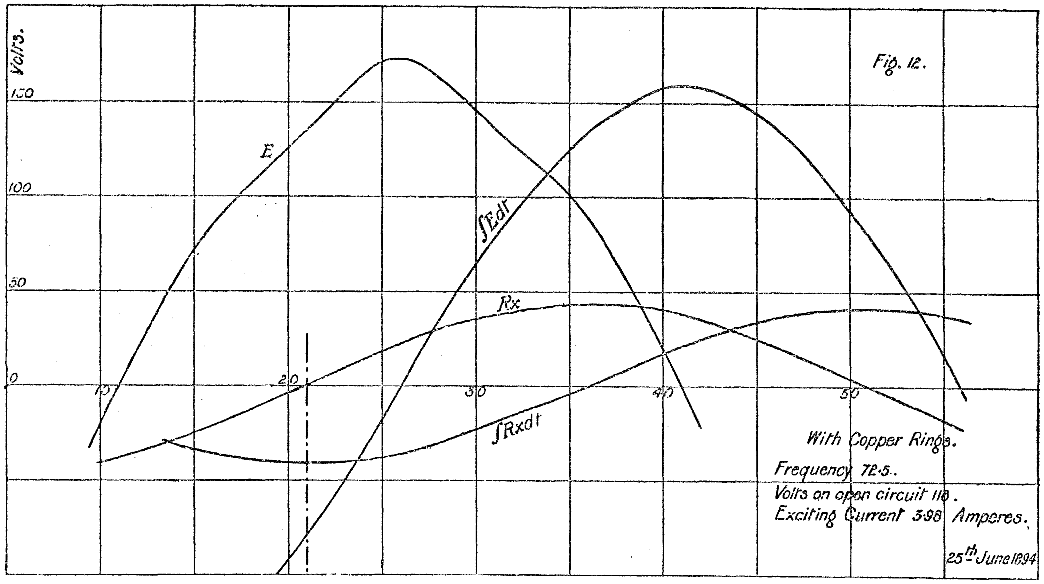
Without Copper Rings.
Frequency 72·5.
Volts on open circuit 117.
Exciting Current 3·9 Amp.
29th June 1894.

the magnets with the copper plates in and the copper plates absent. A comparison shows that the copper plates do not make a great deal of difference. The principal effect is to diminish the current induced in a coil on the magnet placed at position 5, fig. 1, close behind the copper plate.

It may be inferred that in this machine there is conductivity enough in the magnet cores to have in large measure the effect indicated, and that the effect cannot be greatly diminished by compelling the magnetizing current to be constant in the magnet coils, nor can it be greatly increased by exaggerating the currents induced about the magnets by intentionally introducing additional conductivities around them. The effect of each is merely to alter the place where the currents occur.

Recently machines have been built, with finely-divided pole pieces to the magnets by Messrs. MATHER and PLATT and by the BRITISH THOMSON-HOUSTON COMPANY.

Fig. 12.



It was obviously desirable to obtain a verification with a machine of totally different construction. For this purpose we had available the first model made of the alternating machines manufactured by Messrs. MATHER and PLATT. It has an iron core in the armature which projects and extends beyond the armature coils. It was treated in exactly the same way as the SIEMENS' machines but with a fairly full load. The results are shown in fig. 21, from which it will be observed that the total induction actually observed when the machine is loaded is about 11 per cent. below the induction inferred from the electromotive force on open circuit.

II.

The following experiments were primarily made for the purpose of determining the efficiency of the machine, but they will be seen in Section III. to have an important bearing upon the principal subject of this paper.

For the purpose of finding the efficiency of the alternators, when running as generator and motor,[*] the two armatures were rigidly mechanically coupled together,

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[*] This is the same method of test which has been applied to direct current machines (see 'Phil. Trans.,' R.S., 1886, p. 331).

the leading machine being generator, and were connected in series with a non-inductive resistance r, and a KELVIN ampere balance C, as shown in fig. 13. The potential difference of the generator was measured at different epochs by means of the KELVIN quadrant electrometer Q, and the contact maker K, the potential applied to the electrometer being reduced by the non-inductive resistances r_1, r_2. For corresponding epochs a curve of potential was taken across r, this gives the current passing between the machines and also the difference of potential difference between motor and generator.

The power difference, or loss in the combination, was supplied by a shunt-motor through shafting and belts, and was determined by observing the watts supplied to the motor when driving the shafting, the alternator belt being removed, and then observing the watts taken to drive the alternator when loaded—the speed being the same in each case. The difference gives the power absorbed by the combination.

It was found that the watts required to drive the shafting alone were 1681; the watts required to drive the shafting and alternators when excited, but not loaded, were 2479, the difference being in part due to currents induced in the metal frames of the armature.

Tables II. and III. give for about half and full load (with regard to current only) the data for getting at the watts given out by generator and received by motor, and have been obtained from the potential and current curves. The phase difference between the armatures was $\frac{1}{20}$th and $\frac{1}{10}$th period respectively.

Table IV. shows how the efficiencies of generator, motor, and combination have been obtained; and also gives the allocation of losses in the system.

The frequency was about 70 periods per second, and, since the machines are built for 100 periods per second, the figures must be taken only as illustrative of the method of test.

The alternators being connected, as shown in fig. 13, we were able to vary the exciting currents of the two machines by means of the adjustable resistances r_3, r_4. The armatures were coupled $\frac{1}{10}$th of a period apart in phase, and the following experiments were made.

1. The alternators were equally excited with a current of 17·5 amperes and run at a speed of 716 revolutions per minute, corresponding with a frequency of 71·6 periods per second, and the following curves obtained (see fig. 14).

E_G, E_M are the E.M.F.'s of generator and motor when running on open circuit.

PD_G, PD_M are the potential difference of generator and motor respectively when a current of 42·2 amperes ($\sqrt{\text{mean}^2}$) was passing through the armatures.

e_G, e_M are the E.M.F.'s of the respective machines when loaded, that is to say, they are the curves PD_G, PD_M corrected for current into armature resistance.

E is the E.M.F. of the combination when not loaded, that is, it is the difference of the curves E_G, E_M.

x is the curve of current passing between the machines, and is proportional to the

Fig. 13.

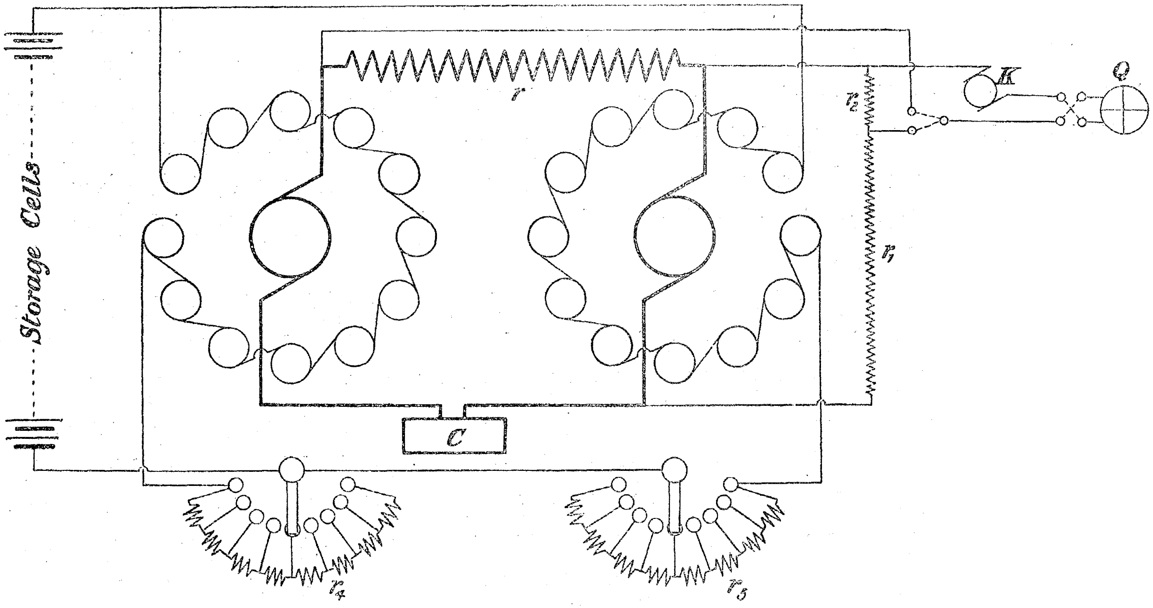

Storage Cells
r
r_2
r_1
K
Q
C
r_4
r_5

Fig. 14.

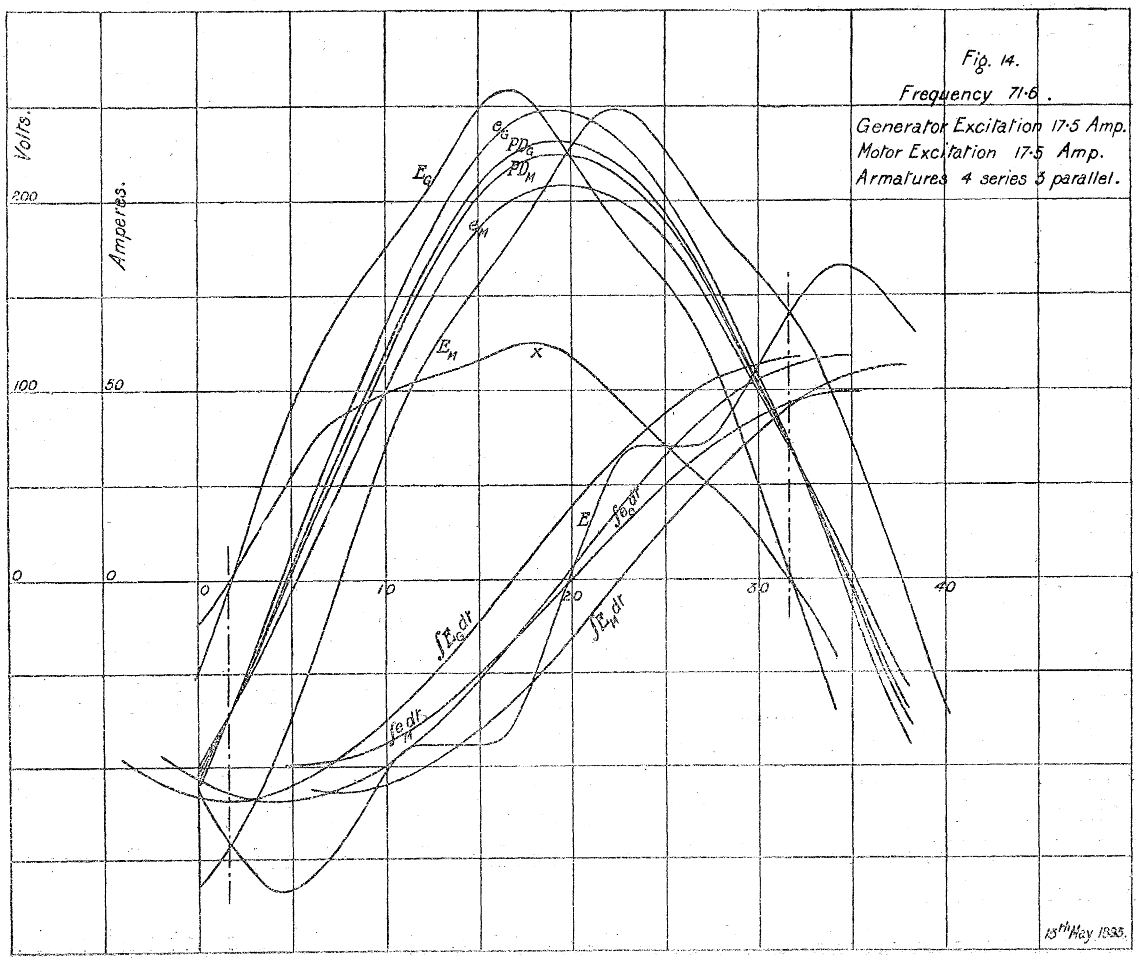


Fig. 14.
Frequency 71·6.
Generator Excitation 17·5 Amp.
Motor Excitation 17·5 Amp.
Armatures 4 series 3 parallel.
Volts.
Amperes.
200
100
0
50
0
0
10
20
30
40
e_G
PD_G
PD_M
E_G
e_M
E_M
X
E
$\int e_G dt$
$\int E_G dt$
$\int e_M dt$
$\int E_M dt$
13th May 1895.

E.M.F (e) of the combination when loaded, the connecting leads being non-inductive. This electromotive force is the difference of the curves e_G, e_M.

2. In fig. 15* the frequency is 71, the motor is excited with 18·6 amperes and the

Fig. 15.

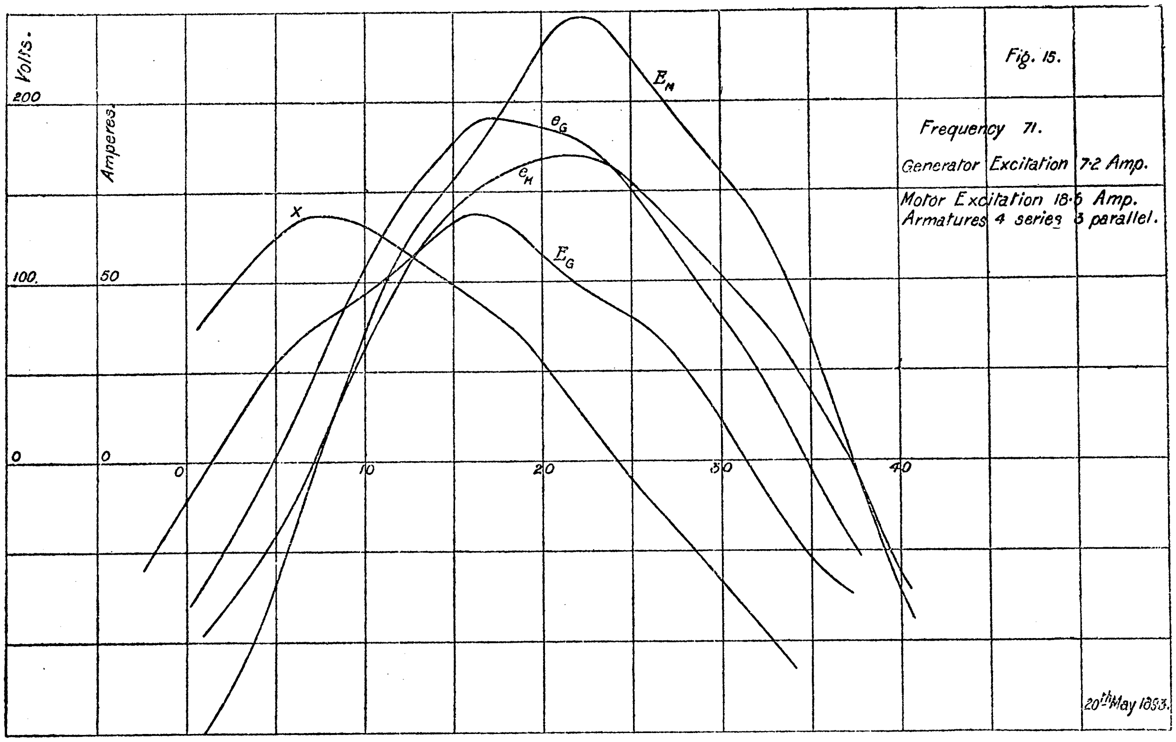
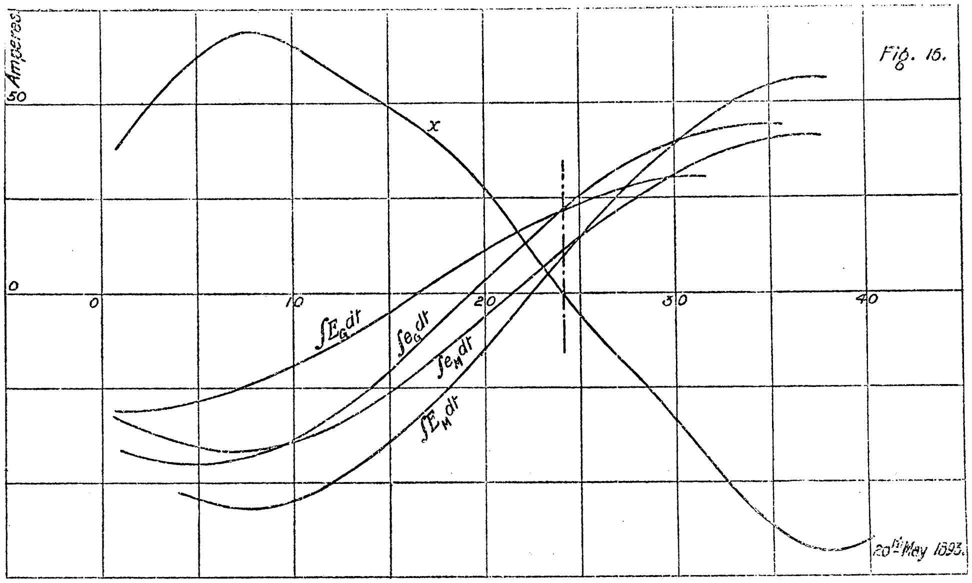


Fig. 16.



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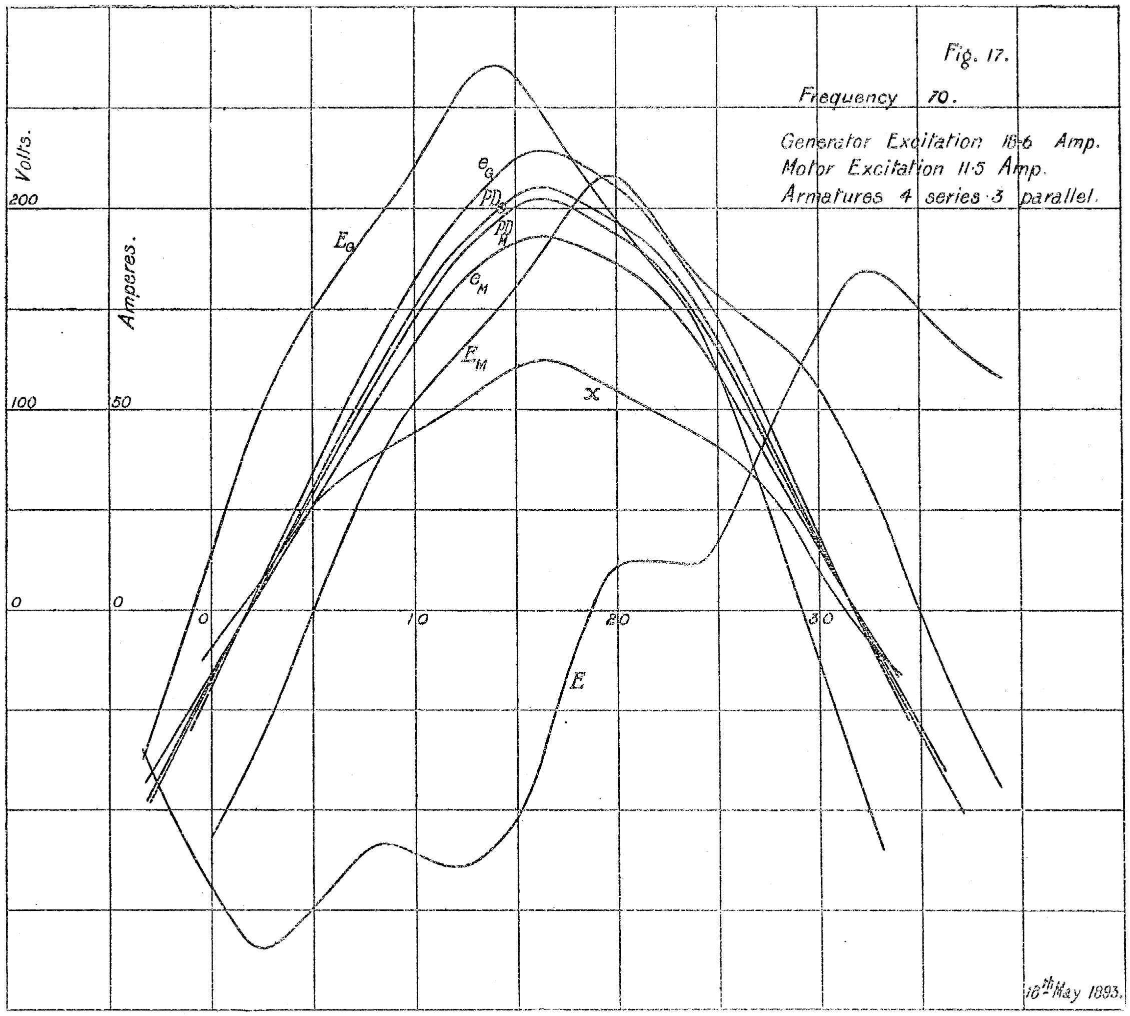
* *Vide* HOPKINSON, Institute of Civil Engineers Lecture delivered 1883; Institute of Civil Engineers, November, 1884; or "Original Papers on Dynamo Machinery," pp. 58 and 148.

generator with 7·2 amperes. In this case the motor is working at higher E.M.F. on open circuit than the generator. Curves corresponding to those in the Experiment 1 were obtained and are marked in a similar manner.

The potential curves in fig. 15 have been integrated, and the integral curves so obtained are given in fig. 16. These give therefore the inductions in terms of the time.

3. In fig. 17 the frequency is 70, the generator is excited with 18·6 and the motor with 11·5 amperes. The generator is working with a higher E.M.F. on open circuit than the motor. The potential curves have been integrated, and the integral curves are given in fig. 18.

Fig. 17.



It was observed that when the exciting current of the motor was decreased, that of the generator being kept fairly constant (the two machines being equally excited with about 18 amperes each to begin with), the current between the machines gradually decreased until a critical point was reached, when a further diminution of the motor exciting current had the effect of increasing the current between the machines.* It was also observed that the watts given out by the generator did not vary in the proportion of the currents between the machines.

Fig. 18.

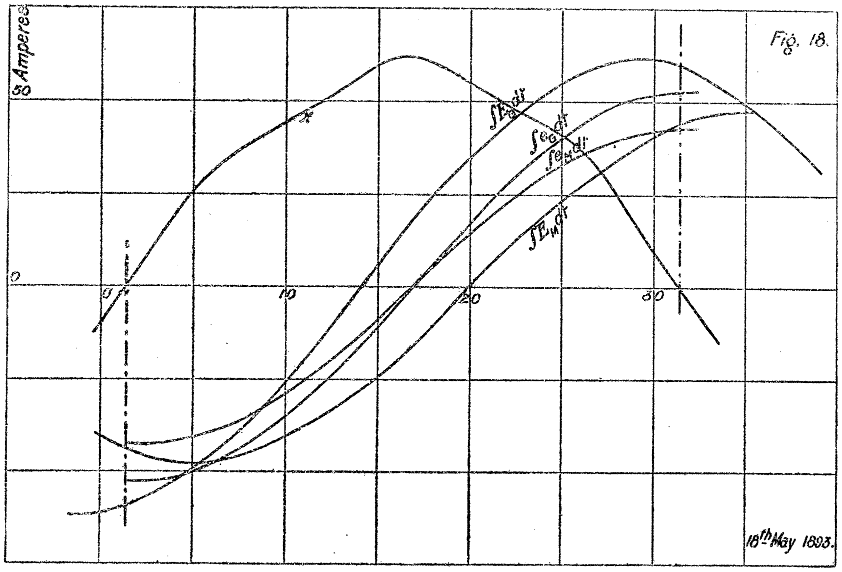


Table V. gives the watts given out by generator for three values of its exciting currents 18·2, 18·6, and 19 amperes, the corresponding exciting currents for the motor being 18, 11·5, and 8·4 amperes. The current between the machines is a minimum for exciting currents of 18·6 and 11·5 in the two machines, and the curves in fig. 17 have been taken under these conditions.

This is a point of practical importance in transmission of power by alternate currents, since the size of the conductor between motor and generator is mainly determined by the current transmitted. The cause is readily explained from the curves.

Starting with the conditions as in Experiment 2, where the motor is more highly excited than the generator, we see that the current (x) is accelerated in phase with regard to the potential difference of the machines. On increasing the exciting current of the generator until the machines are equally excited as in Experiment 1, the current (x) is still accelerated with regard to the potential difference, but not to such an extent. On diminishing the motor exciting current until the machines are excited as in Experiment 3, the current (x) is in phase with the potential difference. For a given power transmitted this will be the point of maximum efficiency with regard to the intermediate conductors. Any further diminution of the motor exciting

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[*] This effect has been independently observed by Mr. MORDEY and Mr. KAPP (see 'Journal of Institute of Electrical Engineers,' vol. 22, pp. 128, 173).

current has the effect of retarding the current (x) with regard to the potential difference, and consequently for the same watts transmitted and the same potential difference the current must be increased. The case in which the conductors between motor and generator have considerable induction or capacity has not been worked out.

The losses in the system can be supplied electrically (instead of by belt as in these experiments) as in the case of direct current machines.*

III.

The results of the last section are valuable in relation to the effects of induced currents in the magnets, the subject of Section I. With a machine working as a simple generator the current lags behind the electromotive force on open circuit by any amount from 0° to 90°. But when a generator and motor are run rigidly coupled together, the current may lead the generator electromotive force, or may lag and the motor may lag by any amount from 90° to 270°. Regarding the relative phases of electromotive force of machines and of current, the machine is a generator when the current is from 0° to 90° behind the machine; it is a motor from 90° to 270°, and again a generator from 270° to 360°. We have already stated that we should naturally expect that the induced currents in the magnets would have little or no effect when E.M.F. and current were in the same phase, and that they would have a maximum effect when the two were 90° apart, or at quarter centres. We should expect further that, as a generator can be made into a motor by reversing the current in the armature, wherever local currents diminish E.M.F. of a generator, they would increase E.M.F. of a motor. That is, we should expect that local currents would diminish E.M.F. from lead 0° to 180°, and increase E.M.F. from 180° to 360°. As a fact, we find this to be partially verified; it seems that local currents diminish E.M.F. from a negative angle of comparative small amount, perhaps 30°, to considerably more than 90°, and that they increase E.M.F. from 180° to over 270°.

Referring to the curves in fig. 14, x and E_G are in phase, and $\int e_G\, dt$ would need increasing 3 per cent. to meet $\int E_G\, dt$, when x vanishes E_M lags 216°, and $\int e_M\, dt$ needs diminishing, that is the currents have increased the E.M.F. In fig. 15 a very small change in the observations would change the character of the results.

We have taken another set of curves shown in figs. 19 and 20. In fig. 19, we have a very small current 3·3 amperes in the generator magnets, and the current is in phase with the generator, the motor being 264° behind the current. The generator is about 12 per cent. low owing to local currents, and the motor is 25 per cent. of its actual value high. To obtain a better standard we excited a machine with 3·3 amperes and passed the same current as before through the armature and an ordinary non-inductive resistance, and found the current lagged 72°, and the E.M.F.

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* See 'Engineering,' 24th March, 1893.

of the machine was diminished 50 per cent. instead of 12 per cent. The results are given in fig. 20.

Fig. 19.

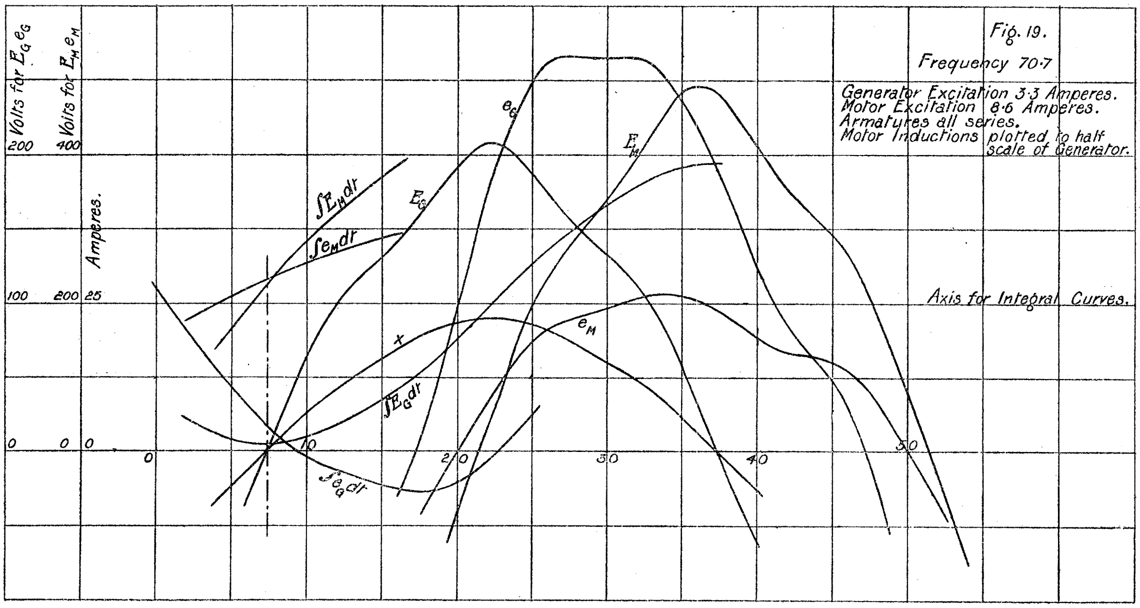

Fig. 19.
Frequency 70·7
Generator Excitation 3·3 Amperes.
Motor Excitation 8·6 Amperes.
Armatures all series.
Motor Inductions plotted to half scale of Generator.

Fig. 20.

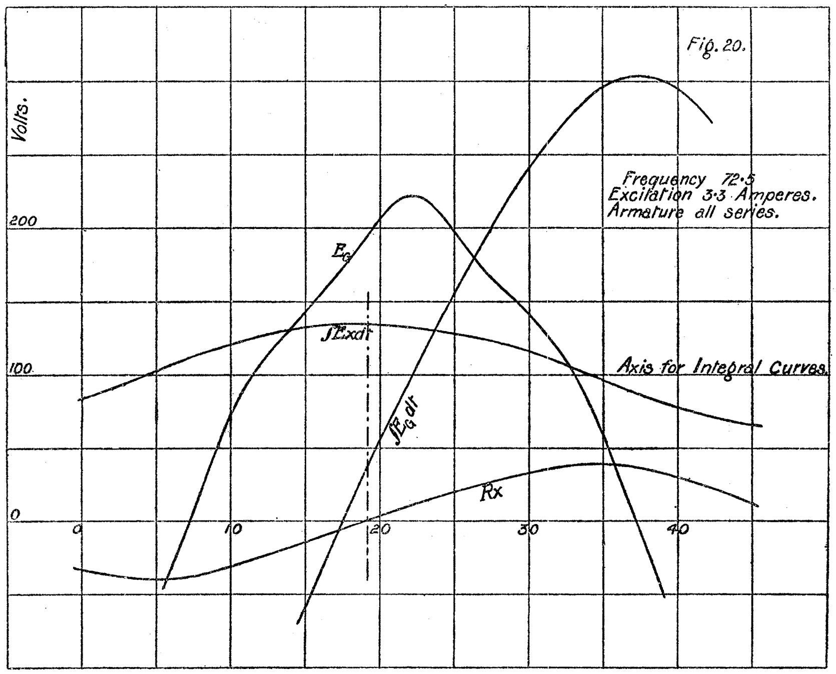

Fig. 20.
Frequency 72·5
Excitation 3·3 Amperes.
Armature all series.

In considering the applications of these results, it must be remembered that the machines have been worked far outside the limits of practice for the purpose of accentuating the effect. If we confined ourselves to these limits we should still find these effects, but smaller in amount.

Fig. 21.

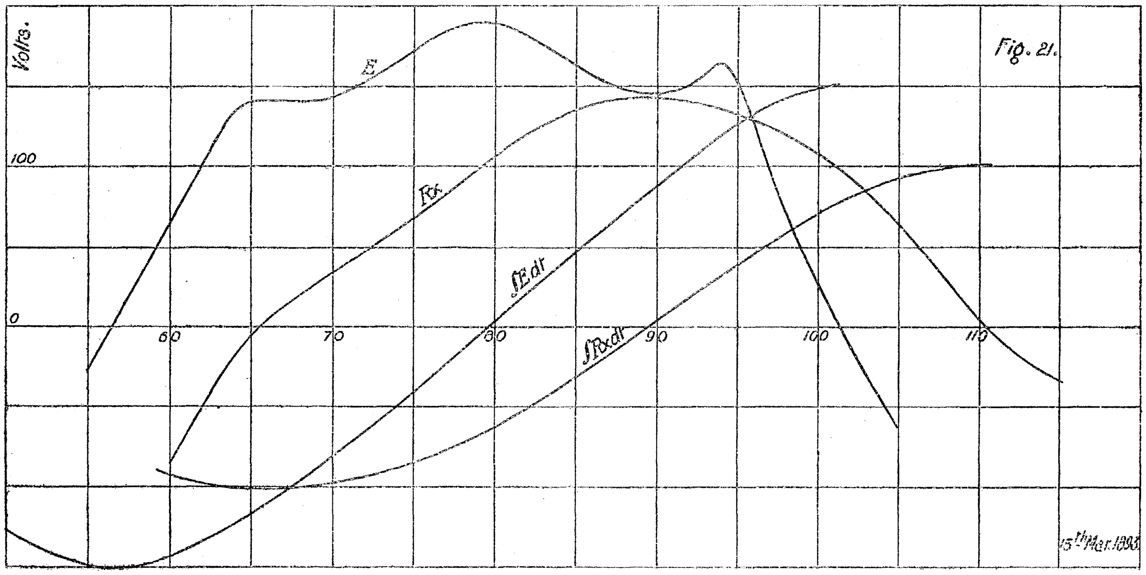

TABLE I.

Fig.	Frequency.	* $\frac{b}{a}$.		Amperes ($\sqrt{\text{mean}^2}$) per armature bobbin.		Exciting current in winding on magnets. Amperes.		Remarks.
		With variations in current in magnet winding.	Without variations in current in magnet winding.	With variations in current in magnet winding.	Without variations in current in magnet winding.	Amperes when normal.	Maximum variation both sides from normal per cent.	
5	92·3	·255	. .	20	. .	8	. .	
	97·7	·149	. .	15	. .	7	. .	
6	95	·13	·147	13·6	13·6	7	22	External resistance altered so as to give same current
6	95	·104	·147	14·1	13·6	7	22	External resistance same in each experiment
7	9·2	·093	·085	8·9	9·1	6	21	External resistance same in each experiment
8	11·7	·05	·035	17·6	17·6	16	20	External resistance same in each experiment

$$* a = \tfrac{1}{2}\int_0^{\frac{T}{2}} \mathrm{E}\,dt - \int_{\mathrm{E}=0}^{x=0} \mathrm{E}\,dt = \text{ordinate of curve A when } x = 0.$$

$$b = a - \tfrac{1}{2}\int_0^{\frac{T}{2}} \mathrm{R}x\,dt = \text{difference of ordinates of curves A and B when } x = 0, \text{ which should be zero on the usual theory.}$$

TABLE II.—Efficiency Test of SIEMENS' W. 12 Alternators. Frequency 70·3 periods per second. Half Load.

Angle in 60ths of a period.	Potential at terminals of generator.		Current between machines.			Potential at terminals of motor.		Watts given out by generator.	Watts given to motor.
	Electrometer deflection.	Volts.	Electrometer deflection.	Amperes.	Sq. of amperes $\sqrt{\text{mean}^2} = 21{\cdot}57$ THOMSON balance } 21·63	Difference of potential difference.	Volts.		
0	−159	−224	+ 96	+23·1	533·6	+2·57	−221·4	−5174	−5114
2	−138	−194·5	+ 76	+18·3	334·9	+2·03	−192·5	−3559	−3523
4	−119	−167·7	+ 62	+14·9	222·0	+1·66	−166·0	−2499	−2473
6	− 99	−139·5	+ 44	+10·6	112·3	+1·18	−138·3	−1479	−1568
8	− 71	−100·0	+ 17	+ 4·1	16·8	+0·45	− 99·5	− 410	− 408
10	− 34	− 47·9	− 18	− 4·3	18·5	−0·48	− 48·4	+ 206	+ 208
2	+ 6	+ 8·4	− 50	−12·0	144·0	−1·34	+ 7·1	− 101	− 85
4	+ 46	+ 64·8	− 79	−19·0	361·0	−2·11	+ 62·7	−1231	−1191
6	+ 79	+111·3	− 95	−22·9	524·4	−2·54	+108·8	−2548	−2491
8	+104	+146·5	−101	−24·3	590·5	−2·7	+143·8	−3560	−3494
20	+124	+174·7	−107	−25·8	665·7	−2·86	+171·8	−4507	−4432
2	+145	+204·3	−116	−27·9	778·4	−3·1	+201·2	−5700	−5614
4	+166	+233·9	−127	−30·6	936·4	−3·4	+230·5	−7158	−7053
6	+179	+252·2	−129	−31·1	967·2	−3·45	+248·7	−7842	−7734
8	+174	+245·2	−116	−27·9	778·4	−3·1	+242·1	−6840	−6754
					6984·1			52402	51726
					465·6			3493	3448

TABLE III.—Efficiency Test of SIEMENS' W. 12 Alternators. Frequency 69·2 periods per second. Full Load.

Angle in 60ths of a period.	Potential at terminals of generator.		Current between machines.			Potential at terminals of motor.		Watts given out by generator.	Watts given to motor.
	Electrometer deflection.	Volts.	Electrometer deflection.	Amperes.	Sq. of amperes $\sqrt{\text{mean}^2} = 42{\cdot}8$ THOMSON balance } 42·0	Difference of potential difference.	Volts.		
0	− 64	− 90·2	+ 25	+ 6·02	36·24	+0·67	− 89·53	− 543	− 539
2	− 35	− 49·3	− 34	− 8·2	67·24	−0·91	− 50·21	+ 404	+ 412
4	− 1	− 1·4	−100	−24·1	580·7	−2·67	− 4·07	+ 34	+ 98
6	+ 32	+ 45·1	−152·4	−36·7	1347·0	−4·07	+ 41·03	− 1655	− 1506
8	+ 63	+ 88·8	−189·5	−45·6	2079·4	−5·06	+ 83·74	− 4049	− 3819
10	+ 94	+132·5	−211	−50·8	2580·8	−5·64	+126·86	− 6731	− 6445
2	+121	+170·5	−222·4	−53·6	2873·0	−5·95	+164·55	− 9140	− 8819
4	+142	+200·1	−236	−56·8	3226·0	−6·31	+193·79	− 11367	−11009
6	+155·3	+218·8	−251·5	−60·6	3672·0	−6·73	+212·07	− 13260	−12850
8	+161·2	+227·1	−251·5	−60·6	3672·0	−6·73	+220·37	− 13763	−13354
20	+160·2	+225·7	−228·2	−55	3025·0	−6·11	+219·59	− 12413	−12080
2	+152·4	+214·7	−188·5	−45·4	2061·0	−5·04	+209·66	− 9746	− 9518
4	+138	+194·4	−147·4	−35·5	1260·3	−3·94	+190·46	− 6902	− 6761
6	+118	+166·3	−113	−27·2	739·8	−3·02	+163·28	− 4523	− 4441
8	+ 92	+129·6	− 75	−18·1	327·6	−2·01	+127·59	− 2346	− 2309
					27548·08 1836·54			96000 6400	92940 6196

TABLE IV.

No.	Description of Magnitude.	Half load.	Full load.
	Frequency in complete periods per second	70·3	69·2
	Phase difference between armatures in fractions of a complete period	$\frac{1}{20}$	$\frac{1}{10}$
1	Watts given out by generator (see Tables II. and III.)	3493	6400
2	Watts given to motor (see Tables II. and III.)	3448	6196
3	Watts dissipated in generator armature = $(\sqrt{\text{mean}^2}\ \text{C.})^2$ ·275 ohm	128·5	481·2
4	Watts dissipated in motor armature = $(\sqrt{\text{mean}^2}\ \text{C.})^2$ ·275 ohm	128·5	481·2
5	Watts dissipated in generator magnet winding	537	537
6	Watts dissipated in motor magnet winding	537	537
7	Watts dissipated in connections between machines	45	204
8	Watts absorbed by combination through belt	1848	2941
9	Total electrical power developed in generator = No. 1 + No. 3	3621	6881
10	Half Watts absorbed by system *minus* half known Watts $= \frac{1}{2}\{\text{No. 8} - (\text{No. 3} + \text{No. 4} + \text{No. 7})\}$	746	889
11	Total power given to generator = No. 5 + No. 9 + No. 10	4904	8307
12	Percentage efficiency of generator $= \frac{\text{No. 1}}{\text{No. 11}} \times \frac{1}{100}$	*71·2*	*77·0*
13	Percentage loss in generator armature	2·61	5·79
14	Percentage loss in generator magnet winding	10·9	6·46
15	Percentage sum of all other losses in generator	15·29	9·75
16	Percentage efficiency of motor $= \left(\frac{\text{No. 9}+\text{No. 10}-\text{No. 8}}{\text{No. 2}+\text{No. 6}}\right)\frac{1}{100}$	*63·2*	*71·7*
17	Percentage loss in motor armature	3·22	7·14
18	Percentage loss in motor magnet winding	13·5	7·98
19	Percentage sum of all other losses in motor	20·1	13·18
20	Percentage efficiency of combination = (No. 12 × No. 16) $\frac{1}{100}$	*45·0*	*55·2*

TABLE V.

Angle in 60ths of a period.	Current in genr. mags. 18·2, motor 18, $\sqrt{\text{mean}^2}$ volts on motor 155. Frequency 70.				Current in genr. mags. 18·6, motor 11·5, $\sqrt{\text{mean}^2}$ volts on motor 143. Frequency 70.				Current in genr. mags. 19, motor 8·4, $\sqrt{\text{mean}^2}$ volts. on motor 132·5. Frequency 70.			
	Volts $\sqrt{\text{mean}^2} = 158$.	Current.		Watts given out by generator.	Volts $\sqrt{\text{mean}^2} = 146$.	Current.		Watts given out by generator.	Volts $\sqrt{\text{mean}^2} = 135·5$.	Current.		Watts given out by generator.
		Electrometer deflection.	Amperes $\sqrt{\text{mean}^2} = 43·3$.			Electrometer deflection.	Amperes $\sqrt{\text{mean}^2} = 40·5$.			Electrometer deflection.	Amperes $\sqrt{\text{mean}^2} = 43·6$.	
0	− 53·3	44	− 9·9	+ 528	− 33·8	42	+ 9·45	− 319	− 23·8	124	+27·9	− 664
3	+ 13·6	155·3	−35·0	− 476	+ 23·3	51	−11·48	− 267	+ 34·4	4	+ 0·9	+ 310
6	+ 75·9	219·4	−49·4	− 3,750	+ 80·3	138	−31·06	− 2,494	+ 81·2	88	−19·8	− 1,608
9	+146·8	249·5	−56·2	− 8,250	+134·2	182·5	−41·08	− 5,513	+127·5	143·5	−32·3	− 4,118
12	+192·6	263·5	−59·3	−11,420	+179·6	222·3	−50·05	− 8,988	+167·5	202·2	−45·5	− 7,622
15	+219·7	277	−62·4	−13,710	+206·2	265·5	−59·76	−12,320	+188·4	263·5	−59·3	−11,170
18	+224·2	251	−56·5	−12,670	+205·2	267·5	−60·22	−12,360	+184·2	282·8	−63·7	−11,730
21	+203·2	188·4	−42·4	− 8,616	+186·6	231·2	−52·04	− 9,710	+165·4	263·5	−59·3	− 9,808
24	+166·6	128	−28·8	− 4,798	+150·6	194·2	−43·72	− 6,584	+132·3	245·5	−55·3	− 7,316
27	+108·6	53	−11·9	− 1,292	+ 96·2	141·5	−31·85	− 3,064	+ 80·4	205	−46·1	− 3,706
				64,454				61,919				57,432